High-order gas-kinetic scheme for fluid structure interaction problems

Kun Xu
Hong Kong University of Science and Technology
email: makxu@ust.hk

Fluid-structure interaction (FSI) problems are important in many engineering applications and are computationally challenging. FSI is a multi-domain problem involving moving and deforming solids which are coupled through solid fluid interfaces. Examples of FSI problems include the modeling of flow-induced vibrations of structures (aircraft, undersea cables, wind turbines, buildings), shock-structure interactions (blast effects), and many others. FSI is an active field of research and there are a wide variety of numerical techniques that have been developed for simulating problems in this area. These techniques include Eulerian, Lagrangian, Arbitrary Lagrangian-Eulerian (ALE), immersed boundary, level-set, interface tracking, and distributed-Lagrange-multiplier/fictitious-domain methods. Numerical approximations based on finite-element, finite-volume and discontinuous Galerkin methods, among others, have been developed.

In this research, we are going to consider to develop a gas-kinetic scheme (GKS) for the FSI problems. The main issue related to the moving mesh method in a physical space is to construct a flux on a moving mesh interface. Suppose reference frame is a rest frame, and an interface is moving with velocity $\vec{U_g}$. In a finite volume scheme, the flux across the interface can be written as below:

$$\mathcal{F} = \int_{\Omega_{\vec{u}_k}} (\vec{u} - \vec{U_g}) f Q(\vec{u}) d\vec{u}, \tag{1}$$

where f is the gas distribution function, $\vec{u}$ is the particle velocity, and Q is the moment to be transported. It is the relative velocity between the particle and interface to determine particle transport across a cell interface. If $\vec{U_g} = 0$, the above formulation goes back to the scheme without mesh movement. The fluxes for macroscopic quantities and distribution function are given as following,

$$\mathcal{F}_{mass} = \vec{n} \cdot \int_{\Omega} (\vec{u} - \vec{U_g}) f d\vec{u},$$
$$\mathcal{F}_{momentum} = \vec{n} \cdot \int_{\Omega} (\vec{u} - \vec{U_g}) \vec{u} f d\vec{u},$$
$$\mathcal{F}_{energy} = \vec{n} \cdot \int_{\Omega} (\vec{u} - \vec{U_g}) \frac{1}{2} \vec{u}^2 f d\vec{u},$$

$$\mathcal{F}_{\vec{u}_k} = \vec{n} \cdot \int_{\Omega_{\vec{u}_k}} (\vec{u} - \vec{U_g}) f d\vec{u}. \tag{2}$$

The local analytical solution of the kinetic BGK equation on a moving interface is

$$\begin{aligned} f(\vec{x}, t, \vec{u}, \xi) &= \frac{1}{\tau} \int_0^t g(\vec{x'}, t', \vec{u}, \xi) e^{-(t-t')/\tau} dt' \\ &\quad + e^{-t/\tau} f_0(\vec{x} - (\vec{u} - U_g)t), \end{aligned} \tag{3}$$

where $\vec{x'} = \vec{x} - (\vec{u} - \vec{U_g})(t - t')$. The initial distribution function f_0 can be constructed through the Chapman-Enskog expansion for the NS solutions around a cell interface. For the integration of the equilibrium part, the corresponding macroscopic variables W_0 are needed. Consider $f_0(\vec{x} - (\vec{u} - U_g)t)$ at $x = 0, t = 0+$, the macroscopic quantities can be determined by conservation constraint,

$$W_0 = \int g_0 \psi d\Xi = \int f_0 \psi d\Xi, \tag{4}$$

where ψ is the conservative moment. Then the equilibrium state and its integration can be determined. The above flux is defined in a local coordinate, if we want to implement the flux in global coordinate, a transformation is needed.

In solid domain we are going solve the elastic wave equation on a fixed reference grid. At a fluid-solid interface, the normal components of the fluid and solid velocities and forces must match along with the fluid and solid interface. The standard partitioned approach to fluid-solid interfaces attempts to decouple these two conditions by choosing the interface velocity to be that of the solid, and the interface force to be that of the fluid. Here we will develop the fluid-solid interface coupled approximation based on the solution of a fluid-solid Riemann problem.